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Supplement: Earth Rotation Measured by a Chip-Scale Ring Laser Gyroscope

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I. MICRO-OPTICAL GYROSCOPE PERFORMANCE

Table I provides a comparison of performance metrics for recently reported micro-optical gyroscopes including the gyroscope described in this work.

Reference	Chip-based (Y/N)	ARW ($^\circ/\sqrt{\text{h}}$)	Bias instability ($^\circ$/h)	Lowest rotation rate measured Ω_{pk} ($^\circ$/h)
12 (SBL-RLG)	Y	N.R.	N.R.	31
13 (SBL-RLG)	Y	N.R.	N.R.	90000
14 (Spiral)	Y	8.5	45	72
15 (RMOG)	N	0.02	3	360†
16 (RMOG)	Y	12	15	36
17 (RMOG)	Y	650	21600	432000
18 (RMOG)	N	N.R.	N.R.	1.3×10^8
This work	Y	0.068	3.6	5

TABLE I. Comparison of the state-of-the-art micro-optical gyroscopes. Reference numbers refer to Main text. †Ref[1]. N.R.: Not reported.

II. EXPERIMENT SETUP

The packaged resonator is shown in Extended Data Figure 1 and the full system setup is shown in the Extended Data Figure 2. An external-cavity diode laser is used as a pump laser and is amplified by an erbium-doped fiber amplifier before being split into two arms. In each arm, the optical frequency is shifted by an acoustic-optical modulator (AOM) and the pump is coupled into the resonator. The pump laser in one arm is phase modulated and locked to the cavity resonance via Pound-Drever-Hall method, while the frequency of the other laser is freely tuned by adjusting the AOM drive frequency. The pump powers are monitored and stabilized by a feedback loop. The pumps and SBLs are combined and photodetected to measure the dual-SBL beating and pump-SBL beating signals. The signals are analyzed using an electrical spectrum analyzer and two frequency counters. The resonator is packaged in a small brass box with fiber connectors. The temperature of the resonator is monitored by a thermistor and controlled by a thermal electric cooler (TEC). Most of the optical components are installed in an isolation chamber to reject acoustic noise, vibration, and airflow. The system is installed on an automated air-bearing rotation stage for the Earth rotation measurement.

III. KERR-INDUCED CONTRIBUTIONS TO THE STIMULATED BRILLIOUN LASER LINEWIDTH

Here we study contributions to the stimulated Brillioun laser (SBL) linewidth that result from the Kerr nonlinearity. The primary noise input is assumed to be thermo-mechanical noise that drives the SBL mode and is responsible for the Schawlow-Townes-like linewidth of the Brillouin laser mode[2]. At room temperature, this noise contribution is nearly 3 orders larger than that from quantum contributions to the linewidth. The dynamics of the SBL can be approximated as follows:[2,3]

$$\frac{da}{dt} = -\frac{\gamma}{2}a + g|A|^2 a + f(t) \tag{S1}$$

where a is the slow-varying amplitude of the SBL in the rotating frame determined by the laser. The square amplitude $|a|^2$ is normalized to photon number, γ is the decay rate, g is the Brillioun gain parameter, A is the amplitude of the pump mode, and $f(t)$ is a Langevin fluctuation term with correlation given by,

$$\langle f(t+\tau)f^*(t)\rangle \approx \gamma N_{\mathrm{T}}\delta(\tau) \tag{S2}$$

where N_{T} are the number of thermal quanta in the acoustic field and a quantum contribution in the normalization is ignored[2]. Lasing occurs when the pump reaches the lasing threshold $|A|^2 = \gamma/(2\mathrm{g})$, and the linewidth of the laser is given by (ignoring quantum contributions),

$$\Delta\omega = \frac{\gamma}{2N_s}N_{\mathrm{T}} \tag{S3}$$

where N_s is the steady-state coherent photon number in the optical mode.

The Kerr effect becomes important when the laser power is increased. To consider this effect we add a Kerr term to the dynamical equation:

$$\frac{da}{dt} = (ig_K N_s - \frac{\gamma}{2})a + g|A|^2 a + ig_K(N - N_s)a + f(t) \tag{S4}$$

where g_K is the single-photon Kerr shift of the SBL mode and $N \equiv |a|^2$ is the laser mode photon number. To estimate the linewidth correction from the Kerr effect, we first linearize the gain saturation near the laser operating point,

$$g|A|^2 \approx \frac{\gamma}{2} + g'(N - N_s) \tag{S5}$$

where $g' < 0$ is the gain derivative with respective to laser photon number N. This expression assumes that the rate of change of the system is slow enough so that the photons in the pump mode ($|A|^2$) adiabatically follow changes in the lasing mode photons (see later discussion relating to eq. S17 below). We can then write down the equations for photon number N and the instantaneous laser frequency fluctuation $\dot{\phi} \equiv (\dot{a}/a - \dot{a}^*/a^*)/(2i) - g_K N_s$ separately (Note: in defining $\dot{\phi}$ we have removed a constant frequency term associated with the Kerr effect):

$$\dot{N} = 2g'(N - N_s)N + (fa^* + f^*a) \tag{S6}$$

$$\dot{\phi} = g_K(N - N_s) + \frac{1}{2i}\left(\frac{f}{a} - \frac{f^*}{a^*}\right) \tag{S7}$$

where the frequency fluctuation equation contains a Kerr effect term that couples the photon number fluctuation $N - N_s$ to the phase noise. For processes slower than the relaxation time scale $1/(g'N_s)$, $\dot{N}$ can be neglected. As a result, fluctuations in the photon number are given by,

$$N - N_s = -\frac{1}{2g'N}(fa^* + f^*a) = -\frac{1}{2g'}\left(\frac{f}{a} + \frac{f^*}{a^*}\right) \tag{S8}$$

Substituting this result into the equation of $\dot{\phi}$ gives,

$$\dot{\phi} = -\frac{g_K}{2g'}\left(\frac{f}{a} + \frac{f^*}{a^*}\right) + \frac{1}{2i}\left(\frac{f}{a} - \frac{f^*}{a^*}\right) \tag{S9}$$

From which the following correlation is readily computed,

$$\langle\dot{\phi}(t+\tau)\dot{\phi}(t)\rangle = \frac{\langle f(t+\tau)f^*(t)\rangle}{2N}\left(1 + \frac{g_K^2}{g'^2}\right) = (1+\alpha^2)\Delta\omega\delta(\tau) \tag{S10}$$

where $\alpha \equiv g_K/g'$ is the amplitude-phase coupling factor and where we replaced N with N_s for steady state operation. We see that the Kerr effect modifies the laser linewidth by a $1+\alpha^2$ factor similar to the well-known Henry linewidth enhancement factor in semiconductor lasers.

This analysis can be readily extended to the beating of two SBLs by introducing cross-phase modulation terms:

$$\begin{aligned}\frac{da_1}{dt} &= (ig_K N_{s1} + 2ig_K N_{s2} - \frac{\gamma}{2})a_1 + g|A_1|^2 a_1 + ig_K\left[(N_1 - N_{s1}) + 2(N_2 - N_{s2})\right]a_1 + f_1(t)\\ \frac{da_2}{dt} &= (ig_K N_{s2} + 2ig_K N_{s1} - \frac{\gamma}{2})a_2 + g|A_2|^2 a_2 + ig_K\left[(N_2 - N_{s2}) + 2(N_1 - N_{s1})\right]a_2 + f_2(t)\end{aligned} \tag{S11}$$

where subscripts 1 and 2 indicate quantities associated with CW and CCW SBL modes, and we have assumed the same gain and loss for the two SBLs. The equations for N_1 and $\dot{\phi}_1$ are now given by,

$$\dot{N}_1 = 2g'(N_1 - N_{s1})N_1 + (f_1 a_1^* + f_1^* a_1) \tag{S12}$$

$$\dot{\phi}_1 = g_K(N_1 - N_{s1}) + 2g_K(N_2 - N_{s2}) + \frac{1}{2i}\left(\frac{f_1}{a_1} - \frac{f_1^*}{a_1^*}\right) \tag{S13}$$

and similar equations exist for N_2 and $\dot{\phi}_2$. We are interested in the phase difference $\theta \equiv \phi_2 - \phi_1$. For rates within the relaxation time $1/g'N_{s1,2}$, it is given by,

$$\dot{\theta} = \frac{g_K}{2g'}\left[\left(\frac{f_1}{a_1} + \frac{f_1^*}{a_1^*}\right) - \left(\frac{f_2}{a_2} + \frac{f_2^*}{a_2^*}\right)\right] - \frac{1}{2i}\left[\left(\frac{f_1}{a_1} - \frac{f_1^*}{a_1^*}\right) - \left(\frac{f_2}{a_2} - \frac{f_2^*}{a_2^*}\right)\right] \tag{S14}$$

Its correlation reads

$$\langle\dot{\theta}(t+\tau)\dot{\theta}(t)\rangle = (1+\alpha^2)(\Delta\omega_1 + \Delta\omega_2)\delta(\tau) \tag{S15}$$

where $\Delta\omega_1$ and $\Delta\omega_2$ are the individual SBL linewidths when the Kerr effect is absent. Again, it can be seen that the noise of the beatnote is corrected by a $1+\alpha^2$ factor.

Now we proceed to find the gain saturation g' parameter, which requires study of the dynamics of the pump:

$$\frac{dA}{dt} = -\frac{\gamma}{2}A - g|a|^2 A + \sqrt{\gamma_{ex}P_{ex}} \tag{S16}$$

where γ_{ex} is the external coupling factor and P_{ex} is the input power to the pump mode. For processes slower than the cavity decay rate ($dA/dt \ll \gamma A/2$), the derivative can be ignored yielding,

$$|A|^2 = \frac{\gamma_{ex}P_{ex}}{(\gamma/2 + g|a|^2)^2} \tag{S17}$$

To eliminate the input term $\gamma_{ex}P_{ex}$, consider the following steady state form of Eq. (S16):

$$0 = -\frac{\gamma}{2}A_0 - gN_sA_0 + \sqrt{\gamma_{ex}P_{ex}}, \quad A_0 = \sqrt{\frac{\gamma}{2g}} \tag{S18}$$

where the pump has its steady-state clamped threshold value (A_0) and N_s is, as above, the steady-state laser photon number. The input term can be solved as

$$\gamma_{ex}P_{ex} = \left(\frac{\gamma}{2} + gN_s\right)^2 \frac{\gamma}{2g} \tag{S19}$$

which, when plugged into the equation for $|A|^2$, gives

$$|A|^2 = \frac{\gamma}{2g}\frac{(\gamma/2 + gN_s)^2}{(\gamma/2 + g|a|^2)^2} \tag{S20}$$

The gain saturation parameter can then be found as,

$$g' = \left.\frac{\partial(g|A|^2)}{\partial|a|^2}\right|_{|a|^2=N_s} = -2g\frac{\gamma/2}{\gamma/2 + gN_s} \tag{S21}$$

Using this expression, the overall dependence of the linewidth on the SBL power can be made explicit:

$$(1+\alpha^2)\Delta\omega = \left[1 + \frac{1}{4}\left(\frac{g_K}{g} + \frac{2g_K N_s}{\gamma}\right)^2\right]\frac{\gamma}{2N_s}N_{\mathrm{T}} \tag{S22}$$

For the silica resonator used here, the SBL gain per photon is $g = 3\times10^{-3}$ Hz, and the Kerr shift per photon is $g_K = 1\times10^{-5}$ Hz. Therefore α is on the order of $10^{-1.5}$ when the SBL power is low, and the correction of Kerr effect to the linewidth is on the order of 10^{-3}.

However, since the magnitude of g' decreases as the SBL power increases, α will increase with SBL power. And eventually, the Kerr-induced noise scales with power and dominates the linewidth. This scaling holds even if we take the quantum noise sources into account. An optimal SBL operating power can be found by minimizing the overall linewidth,

$$N_s = \frac{\gamma}{g_K}\sqrt{1+\frac{g_K^2}{4g^2}} \approx \frac{\gamma}{g_K} \tag{S23}$$

The corresponding linewidth is

$$(1+\alpha^2)\Delta\omega = g_K N_{\mathrm{T}} \tag{S24}$$

which sets a limit to the gyroscope sensitivity. At room temperature $N_{\mathrm{T}} \approx 570$ and the dual-SBL linewidth limit is estimated to be 0.011 Hz (equivalently $0.01°/\sqrt{\mathrm{h}}$ angle random walk). This linewidth is substantially smaller than the one currently measured. Also, this ARW is about $7\times$ smaller than the one currently measured.

We note that the nature of the noise limitation described here is similar to a nonlinear passive resonator as outlined in Matsko *et. al.*[4], where the noise in the nonlinear Kerr frequency shift dominates the frequency noise as the intracavity power increases. The main difference is that the SBL system is thermal-limited, with the vacuum fluctuations replaced by thermal quanta. The single photon Kerr shift g_K can be expressed as

$$g_K = \frac{n_2 \hbar \omega^2 c}{n_0^2 V} \tag{S25}$$

where n_2 is the Kerr-nonlinear refractive index of the resonator material, V is the mode volume, n_0 is the linear refractive index, ω is the angular frequency of the SBL, and c is the speed of light in vacuum. Increasing mode volume and choosing a material with lower n_2 can reduce the Kerr nonlinear effect and subsequently improve the performance limit of the SBL gyro.

IV. OTHER NOISE

Concerning other sources of noise, Kerr-induced cross-phase modulation from pump power fluctuations introduces frequency fluctuations into the two SBL waves that are common mode and hence cancel out when their difference frequency is measured. In addition, power fluctuations from the pump waves will also cause power fluctuations in the SBL waves. These will in turn cause self and cross phase modulation that shifts the difference frequency of the two SBL waves. However, it has been shown that difference frequency shift caused by changes in the SBL powers varies like the difference in the SBL powers[3]. Therefore, because technical power noise from the pump is largely common mode (i.e., both the clockwise and counter-clockwise pump waves are derived from the same laser) it is expected that this noise does not contribute significantly to the beat frequency. Finally, phase noise from the pump can couple into the SBL waves, however, this component of noise can be shown to be weak because the cavity damping rate is much weaker than the phonon damping rate[2].

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[1] Liang, W., Ilchenko, V., Eliyahu, D., Dale, E., Savchenkov, A., Matsko, A. & Maleki, L. Whispering Gallery Mode Optical Gyroscope. *Proc. 2016 IEEE International Symposium on Inertial Sensors and Systems*, 89–92 (2016).

[2] Li, J., Lee, H., Chen, T. & Vahala, K. J. Characterization of a high coherence, Brillouin microcavity laser on silicon. *Opt. Express* **20**, 20170–20180 (2012).

[3] Wang, H., Lai, Y.-H., Yuan, Z., Suh, M.-G. & Vahala, K. J. Petermann-factor limited sensing near an exceptional point. *arXiv:1911.05191* (2019).

[4] Matsko, A. B., Liang, W., Savchenkov, A. A., Ilchenko, V. S. & Maleki, L. Fundamental limitations of sensitivity of whispering gallery mode gyroscopes. *Phys. Lett. A* **382**, 2289–2295 (2018).